PERTH MODERN SCHOOL
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Independent Public School

Course Specialist Year 11

Student name: _____ Teacher name: _____

Date: 22 July 2022

Task type: **Response**

Time allowed for this task: **_____40_____ mins**

Number of questions: **_____6_____**

Materials required: **Calculator-Free**

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: Drawing instruments, templates

Marks available: **__40____ marks**

Task weighting: **__10__%**

Formula sheet provided: **Yes**

Note: All part questions worth more than 2 marks require working to obtain full marks.

Question 1 (2.2.1, 2.2.2) (6 marks)

If $A = \begin{bmatrix} 3 & -1 \\ 1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 2 \\ 5 & 2 \end{bmatrix}$

(a) Determine the matrix X such that $2A - X = B$ (3 marks)

(b) Determine AB (3 marks)

Question 2 (1.3.3) (5 marks)

Let n be an integer. Prove that $n + 2$ is even if and only if $n + 1$ is odd.

Question 3 (1.3.4, 1.3.5) (9 marks)

Write whether each of the following statement is true or false, then prove or disprove it accordingly.

(a) $\forall\, n \in N$, $n^2 - n + 7$ is prime. (3 marks)

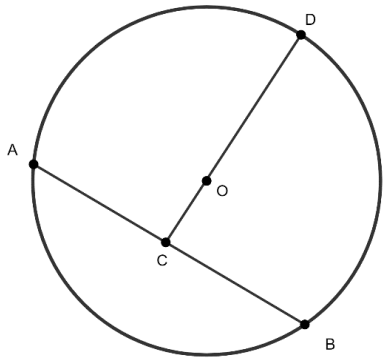
(b) For all irrational numbers p and q, where $p \neq q$, $\frac{p}{q}$ is always irrational.

(3 marks)

(c) There exist two different irrational numbers such that their sum is rational. (3 marks)

Question 4 (1.3.15) (6 marks)

A circle with centre O is shown below (not drawn to scale). Given its radius is r and chord $AB = x$ with $AB \perp CD$, and centre O is on CD



(a) prove that CD bisects AB. (2 marks)
[Hint: Use congruent triangles]

(b) show that $CD = \frac{2r+\sqrt{4r^2-x^2}}{2}$. (4 marks)

Question 5 (2.3.4, 2.3.5) (7 marks)

Use mathematical induction to prove that $\frac{1}{2\times5}+\frac{1}{5\times8}+\cdots+\frac{1}{(3n-1)(3n+2)}=\frac{n}{6n+4}$ for all $n \in Z^+$.

Question 6 (2.3.4, 2.3.6) **(7 marks)**

Use the principle of mathematical induction to prove the following statement:

$3^{2n+4} - 3^{2n}$ is divisible by 5 for all $n \in Z^+$

Additional working space

Question number: _____